# Econ 802

## Second Midterm Exam

Greg Dow November 13, 2019

All questions have equal weight. If something is unclear, please ask.

1. The Acme Company has the input quantities $x = (x_1 \,..\, x_n) \geq 0$ and the input prices $w = (w_1 \,..\, w_n) > 0$. Acme's production function f(x) is strictly quasi-concave and differentiable. Acme minimizes cost for any output $y > 0$, the optimal inputs are always strictly positive, and the prices w are held constant during this question.

(a) Draw a graph showing a U-shaped long run average cost curve. Then use some calculus to show what must be true about long run marginal cost when (i) AC is falling; (ii) AC is at a minimum; and (iii) AC is rising. Draw an MC curve that would be consistent with your results (the graph does not need to be exact).

(b) Prove that the firm's MC at an output $y > 0$ is equal to the Lagrange multiplier for the cost min problem at that output level. Comment briefly on the relationship of this result to the envelope theorem.

(c) Assume that the result in (b) is true. Let $y^* > 0$ and let $x^*$ be the cost min way to produce $y^*$. Prove that $e(x^*) = AC(y^*)/MC(y^*)$, where e(x) is the local elasticity of output with respect to scale. Comment briefly on the relationship of this result to your answer in part (a).

2. Connie has consumption bundles $x = (x_1, x_2) \geq 0$. She has a bliss point at $x^* > 0$. The bliss point gives Connie the highest utility she can ever have. The utility of a bundle x is equal to the negative of the distance from x to $x^*$ (in any direction).

(a) Assume free disposal of goods is <u>not</u> possible. Are Connie's preferences weakly monotonic? Strongly monotonic? Locally non-satiated? Justify your answers.

(b) Choose some $z \neq x^*$ and draw the upper contour set determined by the bundle z. Are Connie's preferences convex? Strictly convex? Justify your answers.

(c) Does Connie always, sometimes, or never have the usual duality relation between utility maximization and expenditure minimization? Explain using graphs.

3. Conrad only cares about two goods: gasoline ($x_1 \geq 0$) and food ($x_2 \geq 0$). Both are normal goods. Conrad has a strictly quasi-concave utility function with local non-satiation. The initial price of gasoline is $p_1 > 0$ and Conrad's initial income is $m > 0$. The price of food is always $p_2 = 1$ (it is held constant during this question).

(a) Politician Y wants to reduce gasoline consumption. She suggests increasing the price of gasoline to $p_1' > p_1$ through a tax, while at the same time raising Conrad's income so that his utility level stays the same as before. Use a graph to explain the effects of this policy. Label axes, intercepts, lines, curves, important points, etc. Will Conrad's gasoline consumption definitely fall? Explain.

(b) Suppose you know Conrad's indirect utility function and his expenditure function. Carefully explain how to find the amount of <u>additional</u> income (beyond his initial income m) he needs to receive from the government under the policy in part (a).

(c) Politician Z points out that it is hard to observe utility. He suggests increasing the price of gasoline to $p_1'$ as in part (a), while at the same time giving Conrad enough income that he can exactly afford to buy his previous consumption bundle. Using a graph, explain whether this policy would reduce gasoline consumption by more or less than the policy in part (a). Which policy would Conrad prefer? Why?

4. Condoleeza has the indirect utility function $v(p, m) = m\,[(p_1/a) + (p_2/b)]^{-1}$ and the expenditure function $e(p, u) = u\,[(p_1/a) + (p_2/b)]$ where $a > 0$ and $b > 0$.

(a) Compute the Hicksian demand functions $h_1(p, u)$ and $h_2(p, u)$. Is the substitution matrix $\partial h(p,u)/\partial p$ symmetric? Is it negative semi-definite? Explain briefly.

(b) Compute the Marshallian demand functions $x_1(p, m)$ and $x_2(p, m)$.

(c) Suppose you try to compute the direct utility function $u(x)$ from the indirect utility function $v(p, m)$ using standard Lagrangian methods. Describe the problems that occur. Do you have a guess about the true $u(x)$ function? Explain.

5. Constantine has the direct utility function $u(c, L) = \ln(c) + \ln(L)$ where $c > 0$ is consumption and $L > 0$ is leisure. The price of consumption is $p > 0$, the wage is $w > 0$, total time is $T > 0$, and non-labor income is $r \geq 0$. Define $m \equiv wT + r$.

(a) Find the Marshallian demand functions $c(p, w, m)$ and $L(p, w, m)$. Then compute the indirect utility function $v(p, w, m)$. You don't need Kuhn-Tucker multipliers.

(b) Suppose that Constantine has zero non-labor income ($r = 0$). How does his labor supply $H = T - L$ respond to changes in the prices $(p, w)$? Provide a clear verbal interpretation of your result.

(c) Suppose there are n consumers who have the same preferences as Constantine but different levels of non-labor income $r_i$ and so different levels of $m_i$. It is possible to transform the utility function in a way that represents the same preferences but leads to the indirect utility function $m_i / 2(pw)^{1/2}$ for $i = 1 \,..\, n$. Will the aggregate consumption and labor supply depend only on p, w, and the aggregate income $M = \sum m_i$ ? Or does income distribution matter? Carefully justify your answer.